MOISTURE DIFFUSION IN MOIST MATERIALS

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UDC 532.72

The first systematic study of the moisture diffusion coefficients of various materials is reported.

The moisture diffusion coefficients must be known to solve certain problems related to temperature and moisture conditions, the design of technological processes, and the practical use of analytic mass transfer solutions.

The mass transfer mechanism (mass in the form of a vapor or liquid) is governed by the nature (physicochemical or physicomechanical) and type of binding of the absorbed material with the solid material and by the thermodynamic conditions for the interaction of the object with the surrounding medium.

Let us consider the experimental data on the dependence of the moisture diffusion coefficients on the moisture content and temperature of the material. A method was proposed in [5] for determining these coefficients under isothermal conditions from the integral drying-kinetics curve:

$$a_m = \frac{0.8NRR_V}{\Gamma(\bar{u}_0 - \bar{u})}. \tag{1}$$

The advantage of this method is that a_m can be determined for a wide range of moisture contents from the results of a single experiment. Study has shown that Eq. (1) can be used to determine the moisture diffusion coefficients in capillary-porous, colloidal capillary-porous, and even in certain colloidal materials.

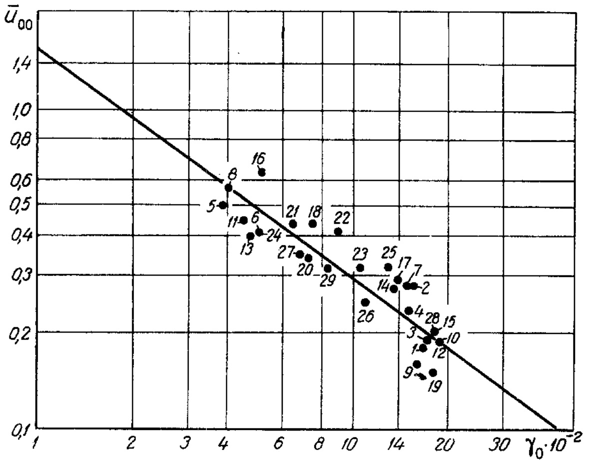

Fig. 1. Dependence of the initial conditional moisture content (kg/kg) for various capillary-porous materials on the density of the materials (kg/m³). The numbers beside the points correspond to the numbers in the first column in Table 2.

A linear dependence of $1/a_m$ on the moisture content follows from Eq. (1). Actually, the function $1/a_m = f(\bar{u})$ is a straight line intersecting the moisture-content axis at the point $\bar{u}_{00} = \bar{u}_{00(gr)}$ and the axis of ordinates at the point $1/a_{m0} \cdot \bar{u}_{00}$; we will call this moisture content the "initial conditional moisture content" since it is not always equal to $\bar{u}_0$ – the actual initial moisture content [5]. The quantity a_{m0} may be called the "conditional moisture diffusion coefficient of the absolutely dry material." The term "conditional" is used because the experimental points deviate from the calculated points at low moisture contents (transfer of the moisture of mono- and polymolecular adsorption).

The numerical value of a_{m0} is found from Eq. (1), which becomes, when $\bar{u} = 0$,

$$a_{m0} = \frac{0.8NRR_V}{\Gamma\bar{u}_0}. \tag{2}$$

Moisture transfer in the material is accelerated as the temperature increases. Experimental studies of

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Institute of Heat and Mass Transfer, Academy of Sciences of the Belorussian SSR, Minsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 15, No. 6, pp. 1019-1026, December, 1968. Original article submitted March 19, 1968.

TABLE 1. Calculated Data for the Determination of a_m of a Porous Ceramic ($\gamma_0 = 1684 \text{kg/m}^3$, R = $0.774 \cdot 10^{-2}$ m)

Material temperature, °C	Drying rate during the period with a constant rate N, kg /kg·h	$1/a_{m0}(\text{gr})$	$1/a_{m0}(2)$	$\dfrac{a_{m0}(2)}{a_{m0}(\text{gr})}$	$\left(\dfrac{273+t}{273}\right)^{20}$	$A_0 \cdot 10^{-5}$
30,0	$1{,}53 \cdot 10^{-2}$	$7{,}40 \cdot 10^5$	$7{,}78 \cdot 10^5$	0,951	8,07	59,7
39,1	$2{,}616 \cdot 10^{-2}$	$4{,}20 \cdot 10^5$	$4{,}55 \cdot 10^5$	0,923	14,50	60,8
47,8	$4{,}941 \cdot 10^{-2}$	$2{,}12 \cdot 10^5$	$2{,}40 \cdot 10^5$	0,883	25,40	53,8

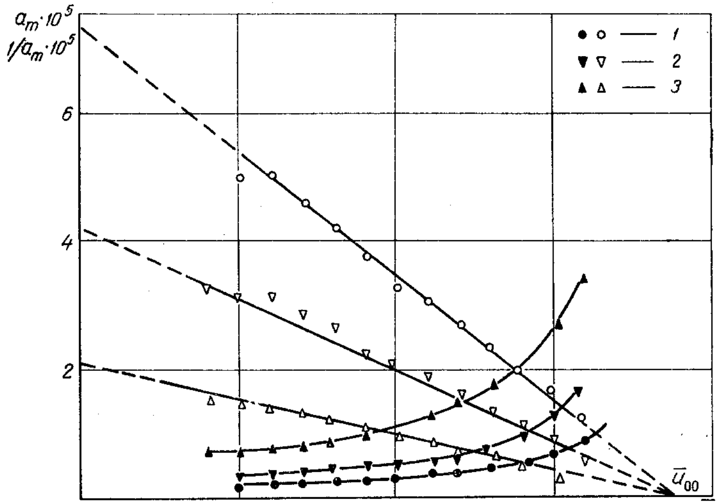

Fig. 2. Dependence of a_m (m^2/h, filled symbols) and 1/a_m (h/m^2, open symbols) on the moisture content (kg/kg) for a porous ceramic at various temperatures: 1) 30.0°C; 2) 39.1; 3) 47.8.

the moisture diffusion coefficients at various temperatures have shown that these coefficients have rather strong dependences on the absolute temperature (they are proportional to T^n) [2, 4, 5, 9, 13, 14, 21, 23].

Accordingly, under the conditions $\bar{u}_0 = \bar{u}_{00}$ and R = const, Eq. (1) can be rewritten

$$\frac{1}{a_m} = A_0 \left(\frac{T}{273}\right)^{-n} \left(1 - \frac{\bar{u}}{\bar{u}_{00}}\right) \tag{3}$$

or, at constant temperature,

$$\frac{1}{a_m} = B_0 \left(1 - \frac{\bar{u}}{\bar{u}_{00}}\right) \cdot \tag{4}$$

There is considerable interest in a numerical determination of $\bar{u}_{00}$. The initial conditional moisture content characteristically decreases with increasing density of the absolutely dry capillary-porous material according to a power law (Fig. 1):

$$\bar{u}_{00} = 51.0\, \gamma_0^{-0.75} \tag{5}$$

or

$$\bar{u}_{00} = B^{-1} \gamma_0^{-0.75}. \tag{6}$$

After the expression for $\bar{u}_{00}$ in Eq. (6) is substituted into Eqs. (3) and (4), the latter become

$$\frac{1}{a_m} = A_0 \left(\frac{T}{273}\right)^{-n} (1 - B\, \gamma_0^{0.75} \bar{u}), \tag{7}$$

$$\frac{1}{a_m} = B_0 (1 - B\, \gamma_0^{0.75} \bar{u}). \tag{8}$$

TABLE 2. Calculated Data for the Moisture Diffusion Coefficients of Capillary-Porous Materials at Constant Temperature

Material	γ_0	t, °C	$\bar{u}$, %	$\bar{u}_{00}$ (5)	$\bar{u}_{00}$ (6)	$\frac{\bar{u}_{00}(5)}{\bar{u}_{00}(6)}$	$B\cdot 10$	$B_0\cdot 10^{-5}$	$a_{m0}\cdot 10^6$	Experimental data of:
1. Natural sand	1660	21	3—15	0,197	0,180	1,094	0,214	0,080	12,500	Murashko
2. Fluvial sand	1539	20	15—26	0,207	0,278	0,745	0,146	0,100	10,000	Huang Fu-Ch'in
3. Silicate brick	1720	22	10—14	0,191	0,184	1,038	0,211	0,280	3,570	The same
4. Sand	1510	20	6—21	0,209	0,235	0,889	0,174	0,300	3,333	Dubnitskii
5. Asbestos – cement slabs	390	20	20—40	0,580	0,510	1,137	0,222	0,480	2,083	The same
6. Diatomaceous slabs	500	20	20—40	0,481	0,410	1,173	0,230	0,540	1,852	" "
7. Sand with particle size 50-120 μ	1530	21	9—24	0,208	0,276	0,753	0,148	0,600	1,667	Murashko
8. Autoclave cellular concrete	400	20	20—40	0,570	0,560	1,017	0,199	0,750	1,333	Dubnitskii
9. Lime mortar	1590	15	6—14	0,202	0,160	1,255	0,246	0,940	1,064	Fokin
10. Refractory fireclay brick	1900	20	8—16	0,177	0,191	0,927	0,182	1,020	0,980	MÉI
11. Cellular concrete	450	20	15—40	0,521	0,448	1,161	0,227	1,590	0,628	Dubnitskii
12. Red brick	1700	15	8—18	0,193	0,192	1,005	0,197	1,660	0,602	Fokin
13. Mixed-binder porous concrete	470	22	20—37	0,502	0,400	1,255	0,246	2,160	0,463	Huang Fu-Ch'in
14. Sand with particle size 6-20 μ	1380	21	13—24	0,225	0,280	0,804	0,158	2,840	0,352	Murashko
15. Silicate brick	1785	15	5—16	0,185	0,200	0,925	0,181	3,740	0,267	Fokin
16. Diatomaceous dust	510	25	40—60	0,474	0,630	0,753	0,148	4,450	0,225	Dubnitskii
17. Sand with particle size 6μ	1380	21	21—26	0,225	0,295	0,763	0,150	4,950	0,202	Murashko
18. Cellular concrete	735	20	10—20	0,362	0,438	0,827	0,162	4,970	0,201	Zhukova
19. 1:2,5:5 Crushed-stone concrete	1790	15	4—12	0,185	0,150	1,232	0,241	5,600	0,179	Fokin
20. Mixed-binder porous concrete	712	20	10—30	0,372	0,340	1,094	0,214	6,190	0,162	Huang Fu-Ch'in
21. The same	642	22	20—40	0,398	0,440	0,904	0,177	7,500	0,133	The same
22. Mixed-binder cellular concrete	885	20	10—35	0,324	0,413	0,783	0,154	8,060	0,124	" "
23. The same	1059	24	20—30	0,274	0,318	0,862	0,169	8,400	0,119	" "
24. Cellular concrete	465	15	13—25	0,507	0,400	1,252	0,247	9,900	0,101	Fokin
25. Porous cement concrete	1285	22	15—24	0,238	0,318	0,748	0,147	14,000	0,071	Huang Fu-Ch'in
26. Concrete with porous clay filler	1101	20	5—23	0,265	0,253	1,046	0,204	20,000	0,050	The same
27. The same	680	20	10—28	0,383	0,350	1,095	0,215	28,200	0,035	" "
28. Ceramics	1684	19,5	4—16	0,194	0,190	1,021	0,200	29,900	0,034	Zhuravleva
29. Cellular concrete	810	15	15—24	0,335	0,320	1,046	0,204	31,000	0,032	Fokin

TABLE 3. Calculated Data for the a_m of Capillary-Porous Materials at Various Temperatures

Material	γ_0	t, °C	$\bar{u}$, %	$A_0 \cdot 10^{-5}$	n	Experimental data of:
1. Sand with particle size 50-120μ	1530	21—40	9—24	1,61	13,0	Murashko
2. Refractory fireclay brick	1900	34—47	8—16	3,30	16,6	MEI
3. Cellular concrete	450	22—45	10—40	3,86	11,0	Dubnitskii
4. Mixed-binder porous concrete	712	20—40	10—30	23,8	19,0	Huang Fu-Ch'in
5. Diatomaceous dust	510	50—65	10—60	40,4	20,0	Dubnitskii
6. Mixed-binder cellular concrete	885	24—34	5—35	51,2	26,0	Huang Fu-Ch'in
7. Ceramics	1684	30—48	4—16	58,1	20,0	Zhuravleva

TABLE 4. Calculated Data for the a_m of Colloidal Capillary-Porous and Colloidal Materials at Constant Temperature

Material	γ_0	t, °C	$\bar{u}$, %	$\bar{u}_{00}$	$B_0 \cdot 10^{-5}$	$a_{m0} \times 10^5$	Experimental data of:
Beet pulp	—	—	40—80	0,88	0,020	50,00	Parfenopulo
Cotton wool	300÷330	40—60	50—90	1,02	0,028	35,71	Shchekoldin
Peat	—	21	50—250	4,00	0,043	23,26	Murashko
Wheat flour, 72%	700	28	12—16	0,21	0,134	7,46	Bab'ev
Sunflower husk	—	—	15—40	0,47	0,500	2,00	Predtechenskii
Complex upland peat, degree of decomposition R = 5%	51	—	800—2000	25,00	0,550	1,82	Korchunov
Woody lowland peat, R = 30%	—	—	300—500	5,30	0,800	1,25	The same
Complex upland peat, R = 20-25%	—	—	400—700	8,20	1,300	0,77	The same
Hemp	—	38	64—337	5,00	1,54	0,65	Khomutskii
Loam	1520	21	12—31	0,33	1,64	0,61	Murashko
Ambary	—	38	83—432	7,00	1,87	0,53	Khomutskii
Clay	1280	21	21—34	0,38	2,65	0,38	Murashko

The use of Eqs. (3) and (4) to determine a_m has been checked for various materials; Eqs. (7) and (8) have been checked for capillary-porous materials (Tables 1-5).

We will now illustrate the practical use of this procedure. We will use the experimental data reported by Zhuravleva on the isothermal drying of porous concrete [8]; a_m was determined from the equation given by Ermolenko [7], so in this case we have all the necessary quantities which appear in Eqs. (1) and (2).

The $a_m = f(\bar{u})$ curves plotted as $1/a_m$ vs $\bar{u}$ at various temperatures level off, intersecting the moisture-content axis at the point $\bar{u}_{00} = 0.19$ kg/kg and the axis of ordinates at several points $1/a_{m0(gr)}$, each of which corresponds to a definite material temperature (Fig. 2).

From Eq. (2), we can determine the method of calculating the intersection $1/a_{m0(2)}$ of the $1/a_m = f(\bar{u})$ lines with the axis of ordinates (Table 1). A comparison of the numerical values of $1/a_{m0(gr)}$ and $1/a_{m0(2)}$ shows that they differ only insignificantly.

For the calculation of the $1/a_{m0(gr)} = f(T)$ dependence (Fig. 2), the power of the dimensionless temperature is assumed equal to 20, so

$$A_0 = \frac{1}{a_{m0\,(gr)}} \left(\frac{T}{273} \right)^{20} = 58.1 \cdot 10^5.$$

Accordingly, in a determination of a_m for a porous ceramic, the following simple dependence may be assumed (within the ranges $0.04 \leq \bar{u} \leq 0.16$ and $30 \leq t \leq 48$):

$$\frac{1}{a_m} = 58.1 \cdot 10^5 \left(\frac{T}{273} \right)^{-20} \left(1 - \frac{\bar{u}}{0.19} \right).$$

Alternatively, if it is assumed that the value $\bar{u}_{00} = 0.19$ kg/kg obtained graphically (Fig. 2) can be found within an error of 2.1% (Table 2) from Eq. (5), then we have

$$\frac{1}{a_m} = 58.1 \cdot 10^5 \left(\frac{T}{273} \right)^{-20} (1 - 0.0196 \gamma_0^{0.75} \bar{u}).$$

TABLE 5. Calculated Data for the a_m of Colloidal Capillary-Porous and Colloidal Materials at Various Temperatures

Material	γ_0	t, °C	$\bar{u}$, %	$\bar{u}_{00}$	$A_0 \cdot 10^{-5}$	n	Experimental data of:
Neva clay	1650	21—32	6—15	0,214	0,61	11,8	Isaev
Osinovskii clay	1820	16—35	6—18	0,264	1,20	8,4	The same
Charsov Yar clay	1850	35—65	10—25	0,320	1,50	6,9	" "
Revda clay	1650	23—35	10—25	0,279	1,65	9,0	" "
Beskudnikovo clay	—	20—38	8—20	0,260	2,63	10,0	" "
Polynkovskii clay	1865	23—42	10—27	0,348	2,90	6,4	" "
Leningrad clay	—	22—35	14—28	0,280	3,26	20,0	" "
Vasyutin clay	1750	20—42	10—25	0,300	4,40	11,1	" "
Loam	1520	21—40	12—32	0,326	5,27	14,0	Murashko
Macaroni dough	—	30—65	30—43	0,500	4000,0	6,0	Orlova [4]

The experimental data on a_m for capillary-porous materials were treated in a similar manner. The results calculated from Eqs. (7) and (8) are shown in Tables 3 and 2, respectively.

The capillary-porous colloidal materials used were clays from various regions and other materials; the colloidal material was macaroni dough.

Tables 4 and 5 show the experimental data calculated from Eqs. (3) and (4) for a_m. It should be noted that all these materials could be treated by dependences (3), (4), and (7), (8) over a wide range of moisture contents (Tables 1-5). Only at low moisture contents was there a discrepancy between the experimental and calculated data.

For practical purposes, the following relation, based on Eqs. (1) and (2), can be used for the dimensionless moisture diffusion coefficient:

$$\frac{a_{m0}}{a_m} = 1 - \frac{\bar{u}}{\bar{u}_{00}}. \qquad (9)$$

Substituting $\bar{u}_{00}$ from (5), (6) into this last expression, we find the following for capillary-porous materials:

$$\frac{a_{m0}}{a_m} = 1 - 0.0196\,\gamma_0^{0.75}\,\bar{u}, \qquad (10)$$

$$\frac{a_{m0}}{a_m} = 1 - B\,\gamma_0^{0.75}\,\bar{u}. \qquad (11)$$

The applicability limits of relations (1)-(11) cannot be specified precisely because of the extreme complexity of the $a_m = f(\bar{u})$ dependence. This complexity occurs because moisture bound in different manners with the solid skeleton of the material separates from the different layers of the material, and the total moisture flux consists of the separate fluxes moving under the influence of various forces.

This procedure for calculating a_m can presumably be extended to the region of physicomechanical binding of moisture and, partially, as experiment has shown, to physicochemical binding; i.e., this procedure applies to the moist and partially hygroscopic state of a material. Studies of 50 different materials confirm that this procedure may be widely used for calculating a_m. There is therefore a good basis for hoping that Eqs. (1)-(11) will find application in the development of methods for intensifying moisture transfer.

NOTATION

a_m	is the moisture diffusion coefficient, m^2/h;
$a_{m0}, a_{m0(gr)}, a_{m0(2)}$	are, respectively, the conditional moisture diffusion coefficient of the absolutely dry material, the same determined graphically, and the same calculated from Eq. (2), m^2/h;
$\bar{u}$	is the mean integral moisture content, kg/kg;
$\bar{u}_0, \bar{u}_{00}, \bar{u}_{00(5)}, \bar{u}_{00(gr)} = \bar{u}_{00(6)}$	are, respectively, the initial (actual) moisture content, the same (conditional), the same [calculated from Eq. (5)], and the same [determined graphically or calculated from Eq. (6)], kg/kg;
N	is the integral drying rate in the initial (constant-rate) period, kg/kg · h;
t, T	are the material temperatures. °C and °K, respectively.
γ_0	is the density of the absolutely dry material, kg/m^3;

n	is the power of the dimensionless temperature;
Γ	is the constant numerical coefficient, equal to three for an infinite plate, four for an infinite cylinder, and five for a sphere [12];
A_0, B_0, B	are the numerical coefficients which are constant for a given type of material;
R	is the characteristic dimension, i.e., the radius of a cylinder or sphere, or half the thickness of an infinite plate, m;
R_V	is the hydraulic radius, m.

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